EX 34 : $z_A = -i \qquad z' = \dfrac{i\bar{z}}{i-\bar{z}}$

1)

$f(M) = M \Leftrightarrow z' = z \Leftrightarrow \dfrac{i\bar{z}}{i-\bar{z}} = z \Leftrightarrow i\bar{z} = iz - z\bar{z} \Leftrightarrow z\bar{z} = i(z-\bar{z})$ *en posant* $z = x+iy$

$\Leftrightarrow x^2 + y^2 = i(2iy) \Leftrightarrow x^2 + y^2 = -2y \Leftrightarrow x^2 + y^2 + 2y = 0 \Leftrightarrow x^2 + (y+1)^2 = 1$

Donc L'ensemble des points invariants par f est le cercle (ζ) de centre A de rayon 1.

2) a) $(z'+i)(\bar{z}-i) = \left(\dfrac{i\bar{z}}{i-\bar{z}} + i\right)(\bar{z}-i) = -i\bar{z} + i\bar{z} + 1 = 1$

b)

$AM'.AM = |z'-z_A|.|z-z_A| = |z'+i|.|z+i| = |z'+i|.\left|\overline{\bar{z}-i}\right|$

$= |z'+i|.|\bar{z}-i| = |(z'+i)(\bar{z}-i)| = |1| = 1$

$\bullet\ (z'+i)(\bar{z}-i) = 1 \Rightarrow z'+i = \dfrac{1}{\bar{z}-i} \Rightarrow Arg(z'+i) \equiv -Arg(\bar{z}-i)(2\pi)$

$\Rightarrow Arg(z'+i) \equiv -Arg(\overline{z+i})(2\pi) \Rightarrow Arg(z'+i) \equiv Arg(z+i)(2\pi)$

$\Rightarrow \left(\vec{u}\,\hat{}\,\overrightarrow{AM'}\right) \equiv \left(\vec{u}\,\hat{}\,\overrightarrow{AM}\right)(2\pi) \Rightarrow \left(\overrightarrow{AM}\,\hat{}\,\vec{u}\right) + \left(\vec{u}\,\hat{}\,\overrightarrow{AM'}\right) \equiv 0(2\pi)$

$\Rightarrow \left(\overrightarrow{AM}\,\hat{}\,\overrightarrow{AM'}\right) \equiv 0(2\pi)$

D'où $\overrightarrow{AM}$ *et* $\overrightarrow{AM'}$ sont colinéaires et de même sens par suite : $M' \in [AM]$

3) a)

$z' = e^{i\theta} \Leftrightarrow \dfrac{i.\bar{z}}{i-\bar{z}} = e^{i\theta} \Leftrightarrow i.\bar{z} = ie^{i\theta} - e^{i\theta}.\bar{z} \Leftrightarrow \bar{z} = \dfrac{ie^{i\theta}}{i+e^{i\theta}}$

$Or\ i + e^{i\theta} = e^{i\frac{\pi}{2}} + e^{i\theta} = e^{i(\frac{\pi}{4}+\frac{\theta}{2})}.\left[e^{i(\frac{\pi}{4}-\frac{\theta}{2})} + e^{-i(\frac{\pi}{4}-\frac{\theta}{2})}\right] = 2.\cos\left(\dfrac{\theta}{2} - \dfrac{\pi}{4}\right).e^{i(\frac{\pi}{4}+\frac{\theta}{2})}$ $\Bigg\rangle \Rightarrow$

$z' = e^{i\theta} \Leftrightarrow \bar{z} = \dfrac{ie^{i\theta}}{2.\cos\left(\frac{\theta}{2} - \frac{\pi}{4}\right).e^{i(\frac{\pi}{4}+\frac{\theta}{2})}} \Leftrightarrow \bar{z} = \dfrac{i}{2}.\left[\dfrac{e^{i\left(\frac{\theta}{2}-\frac{\pi}{4}\right)}}{\cos\left(\frac{\theta}{2} - \frac{\pi}{4}\right)}\right] = \dfrac{i}{2}\left[1 + itg\left(\dfrac{\theta}{2} - \dfrac{\pi}{4}\right)\right]$

$\Leftrightarrow \bar{z} = -\dfrac{1}{2}\left[tg\left(\dfrac{\theta}{2} - \dfrac{\pi}{4}\right) - i\right]$

$\Leftrightarrow z = -\dfrac{1}{2}\left[tg\left(\dfrac{\theta}{2} - \dfrac{\pi}{4}\right) + i\right]$

b) $z^3 = 1 \Leftrightarrow z^3 = e^{i0}$ *les solutions sont* $z_k = e^{i\frac{2k\pi}{3}} \quad k \in \{0,1,2\} \Rightarrow S_C = \left\{1, e^{i\frac{2\pi}{3}}, e^{i\frac{4\pi}{3}}\right\}$

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